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Simulating Gravity: Dark Matter and Gravitational Lensing in the Classroom

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Dark matter makes up most of the matter in the universe but very little of a standard introductory physics curriculum. Here we present our construction and use of a spandex sheet-style gravity simulator to qualitatively demonstrate two aspects of modern physics related to dark matter. First, we describe an activity in which students explore the dependence of orbital velocities on the central mass of a system, in a demonstration of how scientists first discovered dark matter. Second, we discuss the use of the gravity simulator as a visualization of gravitational lensing, a current astronomical technique for mapping dark matter in the sky. After providing the necessary background for the phenomena of interest, we describe our construction of the gravity simulator and detail our facilitation of these two activities. Together, these activities provide a conceptual visualization of gravitational phenomena related to indirect detection techniques for studying dark matter.

Background

Dark matter and galaxy rotation curves

Most of the matter in the universe is invisible *dark matter*, which can only be detected through its gravitational influence. Astronomers know dark matter must be present in galaxies because the galaxies rotate much faster than would be possible if their mass was composed only of things we can see, like stars and dust.

The expected speed of rotation for stars in galaxies can easily be derived using Newtonian physics. Consider a star far from the center of a galaxy that orbits the center in a circular orbit. The force holding the star in orbit is the attractive gravitational force from all the mass inside the star's orbit. Newton's second law gives

$$F = ma_c = G \frac{Mm}{r^2}, \quad (1)$$

where m is the mass of the orbiting star, $a_c = v^2/r$ is the centripetal acceleration of the star, G is the gravitational constant, M is the galactic mass inside the orbit of the star, and r is the distance from the center of the galaxy to the star's orbit.

Solving for the orbital speed of the star yields

$$v = \sqrt{\frac{GM}{r}}. \quad (2)$$

The orbital speed of the star increases with the amount of galactic mass M within the orbit and decreases with increasing distance r . This agrees with intuition: More galactic mass

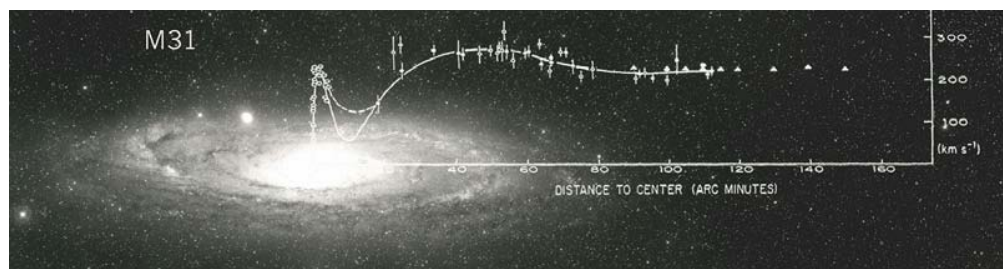


Fig. 1. The rotational velocity for stars in the Andromeda galaxy, M31, as a function of distance from the center of the galaxy overlaid on an image of the galaxy.² Outside of the luminous center, the rotation speed curve is flat and does not decrease as $1/\sqrt{r}$ as would be expected from observations of the luminous matter. (Image by Vera Rubin and Janice Dunlap, courtesy of Carnegie Institution DTM.)

would be able to better hold (faster) stars in orbit, while stars further from the center will feel a smaller gravitational force so that to be gravitationally bound they would need a smaller orbital speed.

If all the mass in galaxies came from luminous matter (matter that interacts with electromagnetic waves, or light), then we would expect the rotation speed of the outer stars to decrease with $1/\sqrt{r}$. In conflict with this expectation, astronomers observe that, outside the luminous center, the rotation speed of stars is approximately constant as a function of their distance from the center of the galaxy (Fig. 1). From Eq. (2), in order for the rotation speed to be constant with r , the total galactic mass M would have to increase linearly with r . Rubin, Ford, and Thonnard write: “The conclusion is inescapable that non-luminous matter exists beyond the optical galaxy.”³ This is dark matter.

Gravitational lensing and the Einstein radius

According to Einstein's theory of general relativity, dark matter exerts a gravitational influence on light particles (photons). This is a manifestation of *gravitational lensing*, wherein light rays are deflected near massive objects. Anything massive can work—a star, a black hole, a galaxy, or a cluster of galaxies all produce lensing. The result is that the images of objects behind these lenses are altered and distorted, and astronomers can measure the lensing to infer the mass of the gravitational lens.

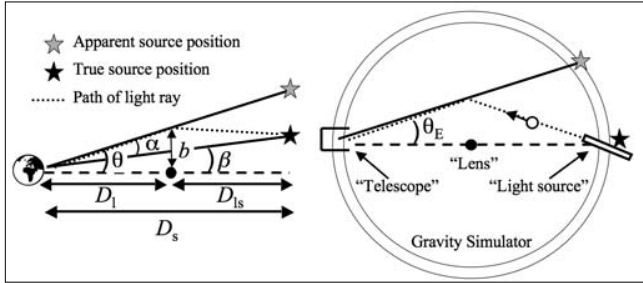


Fig. 2. Generic gravitational lensing geometry (left) compared with the gravity simulator set up for measuring the Einstein radius of the lens (right, described in paper below). The source (represented by the black star) is directly behind the lens (black circle) when $\beta = 0$. In this case, the angle at which the source can be viewed is called the Einstein radius θ_E . For a symmetric lens, the source will be lensed into an Einstein ring.

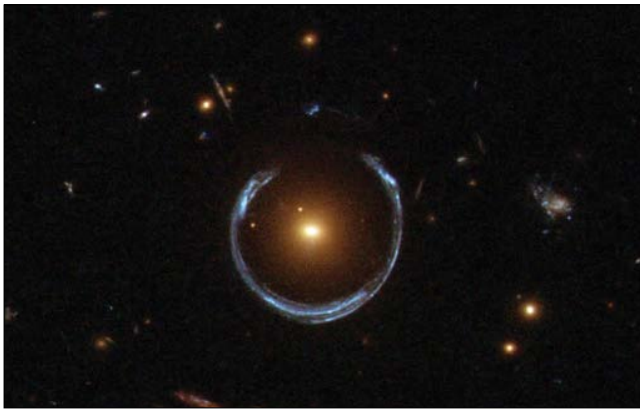


Fig. 3. Example of a nearly full Einstein Ring. (Image courtesy of ESA/Hubble & NASA.)

A lensed background light source at angular position β (relative to the lens) will appear instead to be located at angular position θ . The lens geometry is shown in the left diagram of Fig. 2, yielding the lens equation $\theta = \beta + \alpha$, where α is the deflection angle, given by⁴

$$\alpha = \frac{4GM}{c^2 b} \left(\frac{D_{ls}}{D_s} \right). \quad (3)$$

Here c is the speed of light, we have assumed a point-like lens of mass M , and the other geometrical quantities are defined in Fig. 2. When a background light source is directly behind a symmetric gravitational lens, its light can be projected into an apparent circle around the lens—a phenomenon known as an Einstein ring (Fig. 3). For this scenario we set $\beta = 0$ in the lensing equation and, using $b = D_l \theta$, derive the angular radius of the Einstein ring to be

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{ls}}{D_s D_l}}. \quad (4)$$

A more massive lens (larger M) will deflect light rays more, leading to a larger Einstein radius θ_E .⁵ Astronomers use gravitational lensing to map the dark matter in the sky.

The gravity simulator

The gravity simulator, as conceptualized here, consists of a spandex sheet stretched over a circular frame, similar to a trampoline. Upon placing masses on the fabric, gravitational effects are visualized as a deformation of the fabric. The trajectories of “test particles” (typically small marbles) are curved due to the presence of masses, providing a representation of gravitational effects.

Several papers have investigated the motion of marbles on these spandex sheet-style gravity simulators, finding that the motion does not precisely (i.e., mathematically) reproduce the motion of bodies moving under the influence of gravity.^{6,7} For example, for a spandex sheet that was under no tension before the central mass was put into place, the usual Kepler relation $T^2 \propto r^3$ becomes instead $T^3 \propto r^2$.⁶ However, this demonstration offers a visual and hands-on qualitative representation of gravitational forces which shares some essential features with physical gravitational systems, thus making it a valuable teaching resource for certain conceptual ideas (such as those discussed in this paper).

Building the gravity simulator

In this section, we describe our construction of a gravity simulator. Descriptions of alternative constructions are available.^{8,9}

Our gravity simulator consists of a circular wooden frame over which nylon LYCRA® is stretched. The materials we used to construct the simulator are:

- Plywood
- PVC pipes and joints
- Large sheet of two-way stretch nylon LYCRA
- Spring clamps
- Balls of different sizes and weights (marbles, steel balls, etc.)
- Masses (up to 2 kg)
- Foam pipe insulation (optional)

We used a skill saw to cut the plywood into sections that would form the outside circular edge, before sanding the sections until they were smooth. The wooden frame was glued and screwed together, using wooden braces to join the sections of the circle.¹⁰ The PVC pipes and joints were used to create removable legs for the apparatus. Once the table-like structure was ready (Fig. 4), the nylon LYCRA sheet could be overlaid and held in place by the spring clamps spaced at equal intervals around the border (Fig. 5). It is important that the LYCRA sheet stretches equally in both directions so that the effect of a mass on the sheet is isotropic. The foam pipe insulation can be secured around the outside edge of the gravity simulator to help keep marbles on the simulator and to make the edges safer for participants.



Fig. 4. The circular wooden frame of the gravity simulator. The simulator stands 3 ft tall and the inner diameter is about 5 ft.



Fig. 5. The fully assembled gravity simulator.

Activities

Orbits and galaxy rotation curves

In this activity, students observe that the rotation speed of orbiting marbles is larger for larger central masses.¹¹ This connects with the observations of galactic rotation curves: Since the stars are moving faster than we expect based on observations of luminous matter, astronomers conclude that there is more (non-luminous) matter present in galaxies.

We used an inquiry-based approach, working in small groups (eight) with students. We began by asking students what they knew about gravity, exploring their prior knowledge before making any connections to the gravity simulator.¹² Through facilitated discussion, we encouraged them to ask questions that could be addressed with the gravity simulator. Students are curious but not necessarily good at developing answerable questions; this stage requires some guidance to keep them on topic and within the constraints of the device. We led them to the question of what affects the orbital motion of celestial bodies, and within that to the specific experiment of deciding how orbital speeds depend on the mass in the center of the gravity simulator. Depending on the group, this experiment could look different. For example, the

students might compare the orbital speeds of marbles with two different masses in the center of the simulator or they could observe how the orbital speed changes as they slowly increase the mass in the center.

Students should observe that even if the mass in the center is not directly measurable (as is the case for dark matter), its presence can be inferred and quantified by measuring orbital velocities.

Measuring the Einstein radius

In this activity, students develop intuition about gravitational lensing and qualitatively explore the relationship between the mass of a lens and its Einstein radius.

A “photon source” can be constructed—we used a PVC tube anchored at a fixed angle to the horizontal—which gives the “photons” (represented by marbles) a constant initial speed as they enter the simulator.¹³ A “telescope” (small target container to catch the marbles) is positioned on the opposite side of the gravity simulator, and a lensing mass is placed in the center, as in the right panel of Fig. 2.¹⁴ Students adjust the angle of the photon source until they can hit the telescope, in analogy to actually observing the light from a galaxy behind the lens. Since the lens is approximately halfway between the source and the telescope, the angle of the photon tube is approximately equal to the angle of the photon reaching the telescope. This is the Einstein radius θ_E .

This activity provides a concrete visualization of the path light takes around a massive lens. Students should be able to hit the telescope at the same angle on both sides of the lens, and they should find that larger mass lenses will lead to larger Einstein rings.

Further possibilities

When using the gravity simulator, we emphasized qualitative observations and encouraged students to create their own experiments. In addition to the activities described above, they enthusiastically came up with many “what if?” scenarios to test. One of the most interesting of these was one student’s idea for many of them to simultaneously toss entire handfuls of marbles in different directions around the gravity simulator. This experiment simulates the formation of a solar system or galaxy; as opposing marbles collide and fall to the center, the system is eventually reduced to several marbles (or planets) all orbiting in the same direction.¹⁵

Making connections

To further explore lensing, we supplemented the above gravity simulator activity with a gravitational lensing activity using the glass bases of wine glasses to simulate gravitational lensing effects.^{16,17} Light is bent (or refracted) by the glass, in a similar way to light being bent by gravity near a very massive lens.¹⁷ Experimenting with both gravitational lensing demonstrations in the same session allowed students to visualize the effects of dark matter and gravitational lensing with complementary lower-dimensional representations.

(The gravity simulator removes one of the spatial dimensions in the plane perpendicular to the observer-lens-source direction, while the glass lens demonstration essentially removes the line-of-sight dimension.)

Several other teaching resources are available that focus specifically on dark matter¹⁸ and gravitational lensing,¹⁷ and there are many excellent web resources with additional details on the fascinating mystery of dark matter.¹⁹

The activities described above require an initial investment to build the gravity simulator; once constructed, the device can be used for many years. Although we have described two concrete activities aimed at the introductory physics level, the gravity simulator lends itself to a plethora of different demonstration possibilities and can be used for groups at many different levels²⁰ to vividly illustrate various phenomena related to gravity or general relativity.

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4. James B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Addison-Wesley, 2003).
5. In the universe, changing the distances between observer, lens, and source (which are determined from cosmological redshifts) will also affect the size of the Einstein ring, and the accuracy with which these distances are measured will impact the uncertainty in the final mass estimate. In the gravity simulator, these distances are relatively fixed, as massive lenses naturally sink to the center. The effect of distances on θ_E can be explored using the wine glass demonstration discussed at the end of this article.
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7. Chad A. Middleton and Michael Langston, "Circular orbits on a warped spandex fabric," *Am. J. Phys.* **82**, 287 (April 2014).
8. http://www.huffingtonpost.com/2013/12/04/physics-teacher-dan-burns-gravity-visualized_n_4378960.html.
9. Melissa Hoffman and Meredith Woy, "Fabric of the Cosmos," 2012 Science Outreach Catalyst Kit, <http://www.spsnational.org/programs/socks/2012.htm>.
10. We used plywood to create the circular frame because we found that PVC pipes were too rigid to bend into a circle of our designated diameter. This is in contrast to experience discussed in Ref. 8.
11. Here we used a localized central mass to investigate orbital rotation speeds. A more accurate model of dark matter would be to use an extended object—perhaps a flexible sheet of lead—attached to the underside of the spandex, since the dark matter is known to extend far beyond the visible matter of a galaxy (see Fig. 1). However, our students do not appear to suffer misconceptions about dark matter detection because of this model limitation.
12. In this respect, we started similarly to the activity "Introduction to Spandex as a Model for Spacetime," described in the 2012 Science Outreach Catalyst Kit in Ref. 9.
13. In order to compare the Einstein radius of different lenses, it is important to have a way to standardize the initial speed of the photons.
14. The facilitator may need to remove the optional foam pipe insulation around the edge of the simulator in order to allow marbles to roll off the device and into the telescope bucket.
15. A similar activity, titled "Formation of the Solar System," is described in Ref. 9.
16. Paul Huwe and Scott Field, "Modern gravitational lens cosmology for introductory physics and astronomy students," *Phys. Teach.* **53**, 266–270 (May 2015).
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20. For elementary school students it could be a model of the solar system, while for more advanced students it could be a visualization of Einstein's curved spacetime.

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